2019

Teachers' Learning of Teaching With Multiple Strategies: Understanding Challenges to the Mathematics Florida Standards during a Lesson Study Cycle

Guillermo Farfan, Aki Murata and Alysia Roehrig

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Abstract

Viewing teachers as learners of policy reform, this exploratory study examines a group of elementary mathematics teachers as they discussed teaching with multiple strategies as found in the new Mathematics Florida Standards during a lesson study cycle. In particular, it describes how teachers: (1) advance different explanations for teaching with multiple strategies in the new standards, and (2) anticipate or recognize major obstacles to the implementation of these new standards. Considerations of this study's results to further research on teacher professional development and educational reform are also briefly discussed.

Keywords: multiple strategies, mathematics education reform, lesson study

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In the United States, teaching with multiple strategies in the primary and secondary grades is seen as an important part of reform-based mathematics instruction because they have been shown to facilitate students' mathematical reasoning and demonstration skills (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Sood & Jitendra, 2007). Rather than giving students one strategy to follow, using multiple strategies can help students analyze and contrast different solutions, which in turn may highlight the underlying mathematical concept and thus facilitate students' learning process (Star & Rittle-Johnson, 2008). When properly used, this teaching approach often yields better learning outcomes than traditional instruction focused on mastering procedures and formulas (Pesek & Kirshner, 2000; Star & Rittle-Johnson, 2008).

The Mathematics Florida Standards (MAFS), an offspring of the Common Core State Standards-Mathematics (CCSS-M), is the latest attempt to establish reform-based mathematics instruction in the state of Florida (Blomberg, 2007; Coburn, Hill, & Spillane, 2016). More so than previous efforts, the MAFS are meant to give students "stronger critical thinking, problem solving and communications skills" (Mathematics Florida Standards, 2014) while being mathematically sound (Wu, 2011). Not surprisingly, teaching with multiple strategies is a key component of the MAFS in the elementary and middle grades, reflecting advances on how children develop number sense (Whitacre et al., 2011; see also Ball, 2003; Schoenfeld, 1992). Obviously, the mere presence of multiple strategies in a lesson is insufficient to obtain optimal results: teachers must know how to use them (Silver et al., 2005). Nonetheless, by showcasing multiple strategies as part of the Florida curriculum, the MAFS are setting up expectations for the kind of mathematics instruction teachers are now entrusted to provide.

However, the literature suggests some teachers may have difficulties understanding and teaching the new standards as originally intended (Goodman, 2013; Martin, 2015; Resmovits, 2012), arguably because, at some level, changes in instruction challenge teachers' beliefs of what mathematics is and how it should be taught (Hennessey, 2007). Although the relationship between changes in instructional policy and teachers' reception and subsequent implementation of policy is a complicated one (Coburn, 2001; Olsen & Sexton, 2008), most agree that teachers are key in implementing reform-based policies (Akiba et al., 2016; Durand, Lawson, Wilcox, & Schiller, 2015; Martin, 2015). If that is the case, then more work needs to be done to investigate how teachers—individually and collectively—make sense of reform-based policies and plan to overcome any challenges (Bostic & Matney, 2013).

In this context, work by Lynch and Star (2014) addressed some of the gaps in the literature by looking at middle and high school teachers' views on teaching with multiple strategies. They concluded that secondary teachers had, on the whole, a somewhat superficial understanding of what that entailed (e.g. accommodate different types of learners). Our paper in many ways follows Lynch and Star's (2014); however, rather than based our findings on data collected prior to participating in research-developed summer workshops, we investigate elementary teachers' views while they participated in teacher learning communities, at the time when implementation of the new standards in Florida had just begun.

Accordingly, the aim of this paper is to describe elementary mathematics teachers' responses to the teaching with multiple strategies during a lesson study (LS) cycle. Based on data from interviews and teacher planned meetings, we sought to answer the following: (1) What are teachers' views or understandings regarding the purpose of teaching with multiple strategies in

the MAFS? and (2) What obstacles and supports do they recognize in implementing these new standards?

Theoretical Framework

Teaching with Multiple Strategies

Multiple strategies refer more generally to the process of solving mathematics problems in more than one way, approaching problems from different angles and be able to come up with, compare, and evaluate different solutions (Jitendra et al., 2007; Koedinger & Tabachneck, 1994; Schoenfeld, 1992). In the early grades, the focus is on problem-solving with basic arithmetic (Bergeron & Herscovics, 1990; Verschaffel, Greer, & Torbeyns, 2006; Zazkis, & Mamolo, 2016). Already in the early 1980s, Carpenter and Moser (1984) reported how young children would—without any formal instruction—come up with many counting strategies on their own to solve addition and subtraction problems, such as "counting all" or "counting on." Importantly, students would persist in using many of these strategies even after these strategies were no longer efficient (Carpenter & Moser, 1984). It did not take long to realize that teachers play a significant role in redirecting students to strategies that are more general and promote number sense (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

Despite its long history, teaching children how to effectively use different strategies when solving basic arithmetic problems has remained problematic (Verschaffel, Greer, & Torbeyns, 2006). Part of this may be due to difficulties among some children in transferring mathematical knowledge of basic numbers to solve more complex problems (Kaminski & Sloutsky, 2012; Schoenfeld, 1992; Sood & Jitendra, 2007). For example, a student might be able to correctly decompose 10 as the sums of 1s and/or 5s, but still have trouble using that knowledge to add two-digit numbers such as 15 + 27 (Cheng, 2012; Laski, Ermakova, & Vasilyeva, 2014).

However, equally important may be that teachers are hesitant to spend valuable time using multiple strategies (a time-consuming task) when standard, ready-to-use algorithms are easily available (Borko et al., 1992; Star & Rittle-Johnson, 2008), or they lack understanding of the mathematical principles behind those algorithms to unpack them (Raveh, Koichu, Peled, & Zaslavsky, 2016), or perhaps both.

For this study, teachers who participated in LS all wished to improve their instruction of multiple strategies in early arithmetic. Indeed, the proper use of strategies such as decomposition or "make-a-ten" to the development of later number sense is not only well documented in the literature (Bush & Karp, 2013; Cheng, 2012; Laski, Ermakova, & Vasilyeva, 2014; Schneider & Siegler, 2010), but it also well represented in the MAFS (e.g. MAFS.1.OA.3.6: "Use strategies such as counting on; making ten," Mathematics Florida Standards, 2014). It is this strategy of "make-a-ten" in particular that became the focus of the LS group and that provide much material for discussion among teachers.

Teachers as Learners in Lesson Study

Besides mathematical content, teachers also are learners of policy reform (Spillane, Reiser, & Reimer, 2002). When a new policy is implemented, teachers often gather information about said policy and integrate it with their prior knowledge, beliefs, and understandings, thereby constructing their own interpretations, meanings, and ideas of the policy (Lynch & Star, 2014; Schoenfeld, 1998). Teachers may force these ideas to fit what they know about mathematics instruction, or they may revise or reinterpret their prior knowledge in order to conform with the new policy (Dulude, Spillane, Dumay, 2015; Spillane, 2000). Furthermore, participation in professional development (PD) or learning communities might strengthen or weaken the extent

to which teachers are willing to learn, adopt, and implement a new policy (Cohen & Hill, 2001; Wood, 2007).

In this paper, LS was the medium by which teachers came to learn and discuss teaching with multiple strategies as policy. Originating in Japan, LS is a professional community model where teachers study lessons in depth, which has been linked to better students' mathematics proficiency (Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014). Although there is no one right way to "do" lesson study, in Florida a traditional LS cycle often consist of (1) researching and selecting a topic, (2) writing goals and develop a lesson plan, (3) teaching the research lesson, and (4) reflecting on what happened during the lesson and how it may affect future teacher practice (Gorman, Mark, & Nikula, 2010). All teachers in this study followed this 4-step LS model in their respective schools.

Ideally, LS motivates teachers to analyze and reflect on those aspects of the lesson that best help children's mathematical reasoning (Lin & Rowland, 2016), sometimes guided by a seasoned teacher or LS facilitator. In our case, teachers in the LS group discussed and debated with their LS facilitators a range of decisions at each stage in the cycle, from how to group students to what sort of manipulatives to use for the lesson. This dynamic of discussion and debate is representative of LS, where teachers rely on one another for expertise not only on content knowledge but also for other aspects of teaching and policy that may impact their classrooms (Lewis & Takahashi, 2013), such as those present in the new standards.

Many other characteristics of effective PD, such as peer collaboration, focus on teaching practice and student learning, and responsiveness to local school culture, have also been associated with LS (Lewis, Perry, & Murata, 2006). More importantly, because it is entirely teacher-driven, LS provides an opportunity to observe how understandings of multiple strategies

emerged from teachers themselves, rather than being dictated by or derived from content experts and educational researchers, as it is often the case with other PD platforms (cf. Lynch & Star, 2014).

Method

Setting and Participants

The data were collected as part of a longitudinal LS project that took place in Coast County District, Florida, during the months of January-March of 2016.[1] Two of this paper's authors were members of the research team during the data collection process. Teacher participation in LS and the project was voluntary.

In our sample (N =11), the average teaching experience was 15.2 years; three teachers were relatively new (M =1.3 years of teaching experience) and the rest were experienced teachers (M =20.4 years of teaching experience). All teachers were white, female, and college educated. Four teachers, including two of the new ones, had never participated in a LS cycle before; the remainder had participated in 1 to 13 previous LS cycles (Table 1).

Data Sources

As shown in Table 1, data in this paper come from two sources: individual interviews and LS group planned meetings. In order to obtain a wider range of perspectives beyond the LS group meetings, three teachers and four LS facilitators from five different schools were interviewed. The average time for these interviews was 35 min. The content of the interviews was focused on teachers' experience with LS in addition to questions regarding the new standards and their teaching practice (see Appendix).

Table 1

[1] IRB approval for this project was obtained from the appropriate institutions. To preserve confidentiality, all names of schools, participants, and districts in this paper are pseudonyms.

Participants

	Number schools	Teachers	LS cycles (Total)	Facilitators	LS cycles (Total)
Interviews	5	**Mary**, **Jocelyn,** Ada	3	Marie, Irene, Rachel, Rita	23
Meetings	1	**Chien,** Lise, Barbara, Rosalind	1	Marie	13

Note. New teachers' names (< 2 yrs. teaching experience) appear in **bold.** Marie provided both interview and video data for this study.

Additionally, approximately nine hours of video of LS planned meetings were also transcribed and examined. The LS group consist of four first-grade teachers and one LS facilitator (who was also interviewed) from one of the schools mentioned earlier; most teachers in this group were participating in LS for the first time. The group met five times (for half a day or more) over a two-month period to work on a research lesson. Videos of these five meetings were recorded by the teachers themselves with equipment provided by the research team, which was returned upon completion of the LS cycle.

The content of the LS group's research lesson was decomposing and adding two-digit numbers as per MAFS.1.NBT.3.4 (Mathematics Florida Standards, 2014). The LS group divided the lesson into three tasks: a simple task of "Solve 53 + 4," a more complex task of "Solve 57 + 6" (which requires using the "make-a-ten"), and a more challenging activity where students will add two-digit numbers using a videogame (Minecraft). More importantly, the teachers expected students to be able to see the similarities underlying their different strategies and connect them back to the idea of "make-a-ten."

Data Analysis

We followed an open coding approach for qualitative data analysis (Miles, Huberman, & Saldaña, 2014; Strauss & Corbin, 1998). First, video and interview data were open-coded in

Microsoft Word and Excel. Codes and categories that emerged were then re-evaluated in another cycle of coding to dispense with unrelated ideas (e.g. classroom management, use of manipulatives). This time, paragraphs were bracketed (constituting an "instance") whenever a statement regarding multiple strategies' purpose or their necessity has been made, or when a major obstacle to teaching with multiple strategies or the MAFS has been identified. In addition, we coded for those instances where teachers elaborated on how they benefited from LS, in order to help validate the LS model.

This iterative process ended when no more instances could be obtained from the data. Research findings were then shared with LS Project team members, including one of the principal investigators (who is a co-author) and two research assistants for validation purposes. Any disagreements were resolved through feedback and discussion.

Results

Resulting categories, codes, and number of instances are shown in Table 2. In total, they were 92 instances spread across three main categories: (1) teachers' understandings of multiple strategies in the MAFS ("Multiple Strategies," 55 instances), (2) teachers' descriptions of obstacles to teaching with multiple strategies ("Obstacles," 28 instances), and (3) ways in which participation in LS has benefited teachers ("LS Benefits," 9 instances).

Understanding Multiple Strategies in the MAFS

Teachers offered a number of different explanations of what multiple strategies stand for and what they can accomplish. For instance, when teachers discussed why multiple strategies should be part of mathematics instruction, the most common response was that their inclusion is meant to foment students' conceptual understanding. There are 17 instances of this explanation in the combined data, 12 of which occurred during LS group discussions alone.

Other teacher explanations about why multiple strategies were included in the MAFS included their use to foster students' verbal reasoning and explanation, to help accommodate different types of learners, and to facilitate students' exploration and collaboration skills. Interestingly, the least popular explanations were that teaching with multiple strategies is meant to provide students with a skillset needed for the higher grades, and that they help improve students' arithmetic proficiency (Table 2).

Table 2

Coding Structure and Number of Instances per Category

Category	Codes	Sample	# Instances
Multiple Strategies	Conceptual understanding	"I think that the focus is on students really getting the conceptual understanding…understanding why."	17 (12)
	Verbal reasoning/explanation	"I like that they don't want to just solve the problem, but really explain and analyze what they're doing."	14 (7)
	Accommodate different learners	"[…] their favorite part is when you learn all these strategies, and I said, 'Well, what strategy would you like to use?' That's their favorite. That's usually when they take it on their own."	9 (4)
	Exploration/collaboration	"[…] having all these different strategies, students can help each other figure out what works best for them."	8 (3)
	Arithmetic proficiency	"Also, because some of the strategies built upon each other. You've to learn the basic strategy for addition, or decomposing a number, before the can add double-digit numbers."	5 (4)
	Skillset	"As long as they're building later on…find what works for them. And then, kinda built upon that."	2 (2)
Obstacles	Old epistemic beliefs	"It's a challenge for some teachers because we're used to the old algorithms, and just teaching them in that one way."	15 (8)
	Inconsistent support	"I don't know if it's just our district, or if it's statewide, but we don't have the curriculum or the materials to back it up."	7 (2)

	Developmentally inappropriate	"And some of them mentally are not ready for the strategy that their asking […] They're not."	6 (6)
LS benefits	Meet challenges	"[…] that's something you do in lesson study, as you're writing the lesson, you come up with every possible way a student can solve this problem and think about what the teacher's reaction would be."	7 (2)
	Fill in gaps	"And my [LS] team has helped...talked about what needs to happen, and in what order, so we do talk about that."	1 (0)
	No change	"I don't think it has been much of a change. I think it has been more of the explanation piece that it's probably the biggest change."	1 (0)

Note. Numbers in parenthesis ( ) indicate those instances, included in the total, that occurred during LS planned meetings discussions

A closer look at these instances revealed that the most elaborate explanations occurred during the LS group planned meetings. In these meetings, Marie often exercised a notable influence as she highlighted points for discussion and redirected teachers' attention to aspects of the lesson that may have gone unnoticed. For example, early in the LS cycle, Marie asked her fellow teachers why they should use multiple strategies in their lesson:

Marie: I do think, because we keep talking about different strategies, and I think it's important to get clear on why… why use different strategies? So… if we're teaching children how to add, why do make a ten? "5 + 8 = I'll take a 5 and make 10 plus 3 more," the make a ten strategy. Why don't, "5 + 8 = [counting fingers] 9, 10, 11, 12, 13." Why give them different strategies? So, what's the purpose in it?

Chien: Different types of learners.

Marie: Ok, so we have different entry points because we've different types of learners.

Barbara: Also, because some of the strategies built upon each other. You've to learn the basic strategy for addition or decomposing a number before they can add double-digit numbers in their minds, or their heads.

Here, two ideas seem to be advanced: first, that teaching with multiple strategies is meant to accommodate individual student differences in learning mathematics, and, second, that they are somehow involved in generating later arithmetic proficiency. Yet, Marie's following explanation is rather different:

Marie: And this is important to me. I do different strategies to build number sense. So, it's really building their number sense. Going back to 25 + 8,

> doing that make a ten, they know that 30 is nearby. It's not just "let's do the trick: making a ten!" and everybody has to do this neat trick of making a ten. It's reasoning, it's sitting there and thinking "Ok. I've a 25. What friendly number is nearby?" kind of thing… By using different strategies, it promotes discussion. So, when we're adding 25 + 26, I can take 5 from the 26 and make it 30. And then add 21 to it. Or I might think of 25 + 25, and then add 1 to it—that discussion promotes number sense.

Marie appears to differentiate between knowing to compose/decompose the number 10 (a "neat trick") and strategizing a solution ("sitting there and thinking") that is coherent with a proper number sense (Jitendra et al., 2007; Schoenfeld, 1992; Whitacre et al., 2011). For Marie, the ultimate reason for teaching with multiple strategies lies in their ability to encourage discussion and student reasoning, which in turn leads to a grasp of the concept of number.

This close association between multiple strategies and developing students' number sense only occurred in our data in the LS group's discussions, led by Marie. Even so, not everyone shared Marie's detailed explanation; Chien, at another point, said her biggest issue with using multiple strategies was that "it doesn't matter how they (students) got the answer…if they can figure out what 2 + 2 is, on whatever strategy they use, and they get the right answer, *what's the* problem? (emphasis added)." This sort of response is contrary not only to Marie's but to the reform-based principles behind the MAFS (Mathematics Florida Standards, 2014; see also Coburn, Hill, & Spillane, 2016; Schoenfeld, 2014). Chien's argument, however, is not entirely novel, as similar objections from teachers have been recorded in the literature (Bingolbali, 2011; Silver et al., 2005).

Although no detailed references to students' number sense where found in the personal interviews, teachers did mention other conceptual advantages of using multiple strategies, often tied to students' ability to explain their solutions, as when a new teacher elucidates:

> Jocelyn: My lower students, who don't understand [the standard algorithms], are using all these different strategies now, and they're doing a good job at explaining what they know because they understand the process, whereas some of the high students, using the old algorithms, just know that you crossed out the zero, and it magically becomes ten, because they crossed something else out. They're actually struggling. It's really interesting.

Experienced teachers also highlighted the importance of students providing explanations for their chosen strategies both during group meetings (the "whole point of Common Core[2]" is "for students to explain [the solution] so that somebody else understands"—Rosalind), and personal interviews ("to have an understanding of math that they [students] can explain"… if you can tell to somebody what you've done, you understand what you're doing"—Rachel). Even though these responses lacked the context of shared discussion provided in LS planned meetings, they nonetheless show teachers' awareness of some of the conceptual aspects involved in teaching with multiple strategies.

We note also that teachers' understandings of multiple strategies were not always one-sided or uniform, even among LS facilitators. For instance, Rachel—who earlier highlighted the

[2] A note on language: in interviews and planned meetings, teachers frequently spoke of the MAFS as "Common Core" or otherwise as the "new standards," but never by their official name. The original wording has been kept whenever direct excerpts or quotes are used; the acronym "MAFS" is reserved for discussion of results.

importance of students explaining their solutions—appeared to also endorse the idea of using multiple strategies to support different types of learners:

> Rachel: All of those kids can do math, they just didn't do it the same way. They all got their answers, they all were a success. They all looked like they felt good about themselves. To me, that's a successful lesson.

This is somewhat different from Marie's idea of a successful lesson:

> Marie: These were first graders. They are six years old. And they were talking math extremely well. They were very comfortable in trying to explain their ideas. Very few of them just did [answer]: "Oh, I just did it. It just happened." They were reaching [out] to communicate their ideas…This lesson brought out their ability to communicate and probably helped develop it in some way.

Altogether, this range of responses suggests that teachers' explanations of what multiple strategies stand for and what they can accomplish are not monolithic or uniform. In some cases, they contradict each other, as with Marie's and Chien's views regarding students' number sense. In other cases, such as with Rachel, teachers may entertain more than one view. Given that teaching with multiple strategies can fulfill more than one instructional goal (Silver et al., 2005), this diversity of views is somewhat to be expected.

Obstacles to Teaching with Multiple Strategies

We identified three main obstacles to the teaching with multiple strategies emerging from the data: old epistemic beliefs, inconsistent support, and that they were developmentally inappropriate.

Epistemic beliefs are beliefs and ideas about what mathematical knowledge is and how it should be taught to others (Depaepe, De Corte, & Verschaffel, 2016; Schoenfeld, 2014), and teachers spoke of these beliefs as a major obstacle against teaching with multiple strategies (15 instances). Often, teachers highlighted these old ideas as holding people back from embracing the new standards. For instance, Lise raised the following issue about parents during a LS group meeting:

> Lise: And these ridiculous strategies that are being tested, "well, you know, we're going to test for this strategy, and that's why we're going to do it." That's why all these parents are freaking out, "Why would I use that strategy when I can solve the problem like this?" (emphasis added).

Teacher colleagues too failed to see the point of teaching with multiple strategies (upper grades teachers "are all up in arms, 'Why should I do that, when I can just teach them to align their place values [like this]?"). Other times, teachers compared teaching with multiple strategies against procedural instruction; for instance, Irene expressed how for a lot of her fellow teachers, teaching formulas and algorithms mechanically "is their comfort zone, so that's what they teach, so the students never learn to think through the problem."

As shown in Table 2, apart from conceptually challenging parents' and colleagues' beliefs, another major obstacle to teaching with multiple strategies was inconsistent support from state and district leaders. Tellingly, the overwhelming majority of teachers who mentioned this were experienced teachers. For instance, Rita complained how one year they "were taught [by the district to do] one thing, and the next, it's kinda a whole different [thing]," while Irene said her district was not "providing the curriculum necessary" to meet the standards "in an orderly way" (both Rita and Irene were LS facilitators).

We mentioned earlier that teachers may be disinclined to spend valuable time learning to teach with multiple strategies when other alternatives are readily available; without proper support, this can lead to even more frustration ("we're all inundated, just all the time, so much to think about. And you're going to ask me to think about just one more thing?"—Rita), or even denial ("some teachers are feeling: 'well I was not taught this way, you aren't providing training, so I'm gonna do it the way I've always taught.'"—Marie).

Notably, new teachers did not share this sentiment at all. Instead, they highlighted only the persistence of old teaching strategies (Jocelyn), or had similar reservations regarding parents, who may not know anything beyond the old algorithms (Mary and Chien). Although our sample had considerably more experienced teachers than new teachers (8:3), the fact that new teachers made almost no mention of district leadership or teacher training as a source of strain remains an intriguing finding.

Finally, an obstacle that was mentioned in the planned meetings, but did not surface during the interviews, is that teaching with multiple strategies may be developmentally inappropriate to at least a subgroup of children. However, this objection was always made with particular children in mind, rather than being theoretically motivated (see Lynch & Star, 2014, where a similar finding is discussed; Clements, Fuson, & Sarama, 2017).

Lesson Study Benefits

We noted seven instances where teachers mentioned their participation in LS and how that helped meet challenges of teaching with multiple strategies; for example, how being part of a LS cycle allowed one to share "about these perspectives, understandings, Common Core, and the standards, and how to teach them," or how LS and Thinking Math (another instructional PD) "opened" a teacher's mind to the advantages of teaching with multiple strategies, putting her

"ahead of the game" as it were. In another case, going through LS helped one teacher empathize with her students, by making her receptive to how students go through and learn these strategies on their own ("I think that also helps me, because then I can sort of get the idea, 'Ok, now I know what they [the students] don't get. Now, I know what you don't get.' And so, I can find another way to say it, another way to do it, you know"—Rita). And, at least in one case, participation in LS appears to have assisted teachers filling-in for the gaps in district support (fill in gaps; "And my [LS] team has helped. And the first-grade team sat down together, you know, and talked about what needs to happen, and in what order, so we do talk about that… But it doesn't come from our district" –Ada).

In at least two occasions, teachers in the LS group also spoke of the benefits of LS during their planning meetings. This happened most vividly when teachers were trying to come up with meaningful tasks that will elicit different student strategies. In the following, Barbara and the others have been trying to "Solve 53 + 4" and "Solve 57 + 6" using unit blocks for a few minutes, attempting to anticipate what students might come up with when using "make-a-ten":

Barbara: That's how in their heads, how they'll do it [points to a table]. I don't know how they're going to put it on paper, but that's how they're going to explain to me. I would do it this way—the easy way.

Marie: Well, that would be good. To see our ways, and then you kinda compare. And that's something you do in lesson study, as you're writing the lesson, you come up with every possible way a student can solve this problem and think about what the teacher's reaction would be (emphasis added).

Once again, it was Marie who directed teachers' attention to students' mathematical thinking, in keeping with her role as LS facilitator (Gorman, Mark, & Nikula, 2010; Lewis, Perry, & Murata, 2006).

In sum, by providing a collaborative environment where discussions of student learning can take place, LS helps teachers like Barbara, Rita, and the others not only learn about multiple strategies but also go through the process of thinking how these strategies can be implemented in a lesson.

Discussion

The overall impression that emerges from both interview and LS group meetings data is that teachers mainly find the teaching with multiple strategies conceptually useful, albeit not always for the same reasons. Of these, the idea that teaching with multiple strategies supports students' number sense is the explanation showing the most depth, while the idea that they develop students' verbal reasoning and explanation comes a close second. If anything, these two understandings are aligned with what policymakers and designers of the MAFS and the CCSS-M had in mind (Clements, Fuson, & Sarama, 2017; Schoenfeld, 2014; Zimba, 2016), and probably reflect elementary teachers' historical familiarity with reform-based content in the United States (cf. Lynch & Star, 2014).

However, underneath the surface, there were other explanations that were more difficult to read. For instance, the idea of using multiple strategies to accommodate different types of learners could be interpreted as attending to student needs along their learning progressions or trajectories (Empson, 2011; Lobato & Walters, 2017). Or it could be interpreted as resembling the controversial idea of learning styles, as some have argued in the past (Lynch & Star, 2014; Pashler, McDaniel, Rohrer, & Bjork, 2008). If teachers are mixing what they know about

multiple strategies with ideas found elsewhere, then more needs to be done to clarify how that may interfere with appropriate policy implementation (Spillane, 2000; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005). Even so, the fact that teachers are learning their way through these different understandings—particularly in the LS planned meetings—instead of blindly accepting any authorized version from the onset, should actually be seen as a good sign (Dulude, Spillane, & Dumay, 2015).

Elementary teachers in this study also identified two main sources of strain or obstacles to teaching with multiple strategies as a change in policy: First, how multiple strategies are challenging parents' and teachers' prior beliefs, and second, the inconsistent support from state/district leadership when it comes to school and classroom implementation. There is no denying that teaching with multiple strategies as per the CCSS-M represent a more rigorous approach to mathematics instruction, and there has been widespread mistrust and misunderstanding in the United States regarding how these new policy changes in instruction are meant to be realized (Wu, 2011; Zimba, 2016). Our results showed that, on a few occasions, teachers shared some of the same objections that the general public may have regarding using multiple strategies to teach mathematics, such as not being developmentally appropriate or not being necessary so long as the answer is correct (Clements, Fuson, & Sarama, 2017). It must be noted, however, that neither experienced nor new teachers outright opposed teaching with multiple strategies or the MAFS, even if there were some who had reservations.

On the other hand, this study did find instances where teachers expressed dissatisfaction with the district and state leadership. In this case, the subtle but important difference between experienced and new teachers' responses during interviews suggest different frames of reference may have been accessed when answering these questions (Spillane, Reiser, & Reimer, 2002).

Conceivably, experienced teachers' prior experience with policy implementation may have played a role in their meaning-construction of the new policy (Lynch & Star, 2014; Spillane, 2000), which new teachers did not have. Although more research is needed to see how widespread this sentiment is among other schools and districts, nonetheless, the fact that the sentiment exists should be a sign of concern. If indeed teachers are key in implementing reform-based policies as intended (Martin, 2015), then districts should pay more attention to what their teachers are thinking and saying—particularly experienced teachers, who might have to change the most (Schoenfeld, 2014).

Finally, we found some limited but encouraging evidence that participating in LS has helped some teachers in their learning of what teaching with multiple strategies entails (e.g. during the LS group discussions led by Marie). Identifying those PD efforts that support teacher learning opportunities and focused on students' sense-making may be another way to ensure teachers learn to implement reform-based policies in the way that was intended (Borko, 2004; Bostic & Matney, 2013).

Limitations

Given its exploratory nature, there are a number of limitations to this study. First, participation in LS and the project was voluntary, meaning teachers in this study were self-motivated to become part of LS. It is conceivable, then, that they were also more self-motivated to learn about multiple strategies and the MAFS than their non-participant peers. Although we widened the net of participating teachers to include people from more than one school in the district, their views and ideas expressed here might still not be representative of the teacher population in Florida as a whole.

Second, even with a relatively small sample, there were more experienced teachers (measured by years of teaching experience) than new teachers—though most were also relatively new to LS. This was not by design; it is quite possible that new teachers, still unfamiliar with their many responsibilities and trying to make a good impression (teachers are no longer tenured in the state of Florida) will be more hesitant to make the time commitment necessary, including time away from their classrooms, to participate in LS.

Finally, although videos of classroom teaching were captured during the project's duration, none of those data have been used here, as they are still in the early process of analysis. It remains to be seen how closely teachers' deliberations here resemble their own teaching with multiple strategies. As Lynch and Star (2014) noted, such data are sure to provide an additional perspective for understanding teachers' views on multiple strategies, beyond what can be captured during interviews and LS group meetings.

Conclusion

The findings of this study should help generate new discussion on how to help teachers educate and train themselves in teaching with multiple strategies. The advent of new standards such as the MAFS in the United States requires that teachers teach many mathematical topics in an unfamiliar way (Wilson & Downs, 2014). This study's results suggest that, although most teachers seemed willing to teach with multiple strategies, their views on what teaching said strategies entail, or what they were set to accomplish, were not always uniform or consistent.

It is helpful, therefore, to recognize that teachers are also learners of policy reform, and not expect that they will know how to adopt and interpret new reform-based policies merely by making information on them available. Doing so ignores the conceptual and pedagogical challenges the new standards represent, as well as their interactions with colleagues, their school

and district leadership, and their prior experiences with PD. Rather than simply requiring teachers to "know the standards" or "learn more math," district and school leaders would do well in identifying PDs, such as LS, that provide a forum for teachers to learn how to implement multiple strategies to foment student learning.

Acknowledgments

This study is based upon work supported by the National Science Foundation under Grant DRL-1417585. Any opinions, findings, and conclusions expressed in this paper are our own and do not necessarily reflect the views of the National Science Foundation.

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Appendix

LS Facilitator and Teacher Interview Protocol

Facilitator question

1. What were your goals LS this year? How do you feel you accomplished these goals this year? [Invite as much details and explanations as possible]

Schedule, Content Focus, and Group

2. How did the schedule work for you? [make sure we understand what schedule this group used] What changes would you suggest in scheduling the meetings?
3. What do you think about the focus on the content? What was useful about focusing on this topic for you? What was challenging about this topic for you?

Planning Meetings and Resources

4. Could you tell me 2 or 3 things you liked about these planning meetings?
5. Was there anything that you felt could be improved or changed about these planning meetings?
6. [if Knowledgeable Other was present] What did you think about having a knowledgeable other? In what way it was useful? What can be improved?
7. What did you like about [facilitator]'s approach to facilitating lesson study? What suggestions would you offer her?

Research Lesson and Debriefing

8. In observing (or teaching) students, what did you notice about their responses to the lesson? Did you notice anything you did not expect?
9. How do you think the problem used in this lesson helped students learn the concept?

10. What did students find challenging in learning the concept in the lesson? [ask for specific data/descriptions]
11. [if revised lesson was taught] Did the second lesson address the particular challenges? Do you think the revisions made a difference in how students learned?
12. What would you do to address the challenges if you are teaching this lesson again?
13. What did you learn from the debriefing session about teaching and student learning in this research lesson?
14. How were the data shared by the group members useful for you to understand students' responses to this lesson?

Your Learning through Lesson Study

15. Now that you completed a cycle of lesson study, how do you think your impression of lesson study changed, if any?
16. What are your biggest takeaways from going through this lesson study cycle?
17. What did you learn about collaborating with other teachers through this lesson study?
18. How did your knowledge and ideas about the content topics change?
19. How did your ideas about student learning of the content topics change?
20. How did your ideas about teaching the content topics change?
21. Would you participate in lesson study again in the future?
    a. [If yes] Would you invite other teachers to participate in lesson study in the future? In what ways do you think we can help other teacher become interested in lesson study?
    b. [If yes] Would you be continuing to facilitate lesson study in the future?

c. [if no] Would you provide reasons why you do not want to participate again? Do you have suggestions for making lesson study more doable and exciting for teachers?

Teaching Practice and Mathematics Florida Standards

22. Recently, there has been a push in Florida to teach math differently, with an emphasis on teaching with multiple strategies in the new standards.
    a. As a teacher, what are your thoughts on that?
    b. Do you see any significant obstacles or challenges?
    c. How would you say your experience and knowledge of mathematics have prepare you for these changes? What about outside support [district, etc.]?
23. Let's say you want to teach your students how to add 13 + 9 =, which requires to 'carry the 1.'
    a. Could you tell me how would you teach that particular problem [ask for any specific strategies]?
    b. What signs or evidence would you use to know if your students understood the concept?
24. Suppose now that I'm one of your students, and you ask me to subtract 23 – 4. After some thinking, I tell you the answer is 21, rather than 19.
    a. How would you respond?
    b. In your view, what is the best way to handle students' mistakes?